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I am submitting herewith a dissertation written by Kevin E. Burns entitled “Ground Penetrating Radar Investigations on the Relationship between Horizontal Sub-wavelength ‘Thin-layer’ Bedrock Fractures and Reflection Amplitudes.” I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Geology.

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(Original signatures are on file with official student documents.)
Ground Penetrating Radar Investigations on the Relationship between Horizontal Sub-wavelength ‘Thin-layer’ Bedrock Fractures and Reflection Amplitudes

A Thesis Presented for the Master of Science Degree
The University of Tennessee, Knoxville

Kevin E. Burns
May 2008
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Dedication

For my wife, Elizabeth M. Burns, thank you for all the love, support, and patience throughout this process. You are the best.

This thesis is also dedicated to the Burns family, the Jones family, to the faculty members at the University of Tennessee and the University of Texas, all of whom played key roles in my life and education.

Hook ‘em Horns and Go Vols!
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Abstract

Several theoretical equations that predict sub-wavelength ‘thin-layer’ reflection amplitudes are compared to the results of a series of controlled ground penetrating radar surveys using 1 GHz transducers over a physical model of a horizontal bedrock fracture. Two large plastic (UHMW-PE) blocks, separated by one or more stacked inserts (polyethylene; ~0.1 mm thick) for a total of 101 surveys, generate a modeled fracture with an aperture ranging from 0-300 mm. All existing theoretical reflection coefficient equations fail to predict observed reflection amplitude oscillations in the data when the fracture aperture is less than 1/48 of a wavelength. The only theoretical formulation to properly predict any significant aspect of the fracture EM reflectivity is the Widess equation; however, the best fit only occurs where aperture sizes are less than 1/16, not 1/8 of the wavelength as predicted. Thermal expansion and temperature fluctuations do not sufficiently account for the oscillations.

The influence of salinity on a water-filled sub-wavelength constant aperture (5 mm) fracture using 1 GHz antennas is also investigated. Results indicate that at this frequency, the reflection amplitude has a slight negative correlation with changes in salinity from 0-5700 mS/m.
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1.0 Introduction

1.1 Motivation

Non-invasive in situ fracture characterization currently presents a difficult problem for modeling groundwater flow and contaminant fate and transport. There are several different methods employed to characterize fractures such as examining surface expressions of fractures in bedrock outcrop or examining the fracture properties from core samples or down boreholes. But these techniques are not without an inherent flaw: fractures in the core sample, in the edge of a borehole, or at the surface cannot be reasonably assumed to have the same characteristics at depth because of increased confining pressure and natural geologic variability. Some hydrogeologists bypass this step altogether by installing monitoring wells and back calculating the properties of the bedrock, assuming that the borehole they have installed captures a representation of all fractures throughout the region of interest. This assumption is likely inadequate, and consequently there is a need for alternative methods. The importance of accurately characterizing fracture geometry is illuminated with examination of the models for fluid-flow in fractured media. Parallel plate, equivalent porous media, discreet fracture, theoretical models, and double-porosity models are common methods. The cubic law (Lamb, 1932) can be found within many models (e.g. parallel plate modeling by Snow 1968, Snow 1969) and demonstrates the accuracy needed for good modeling, i.e., doubling the aperture size results in an 8-fold increase in discharge. Surface ground penetrating radar (GPR)
surveys present an attractive alternative because the full 3D fracture distribution may be qualitatively determined \textit{in situ} (Talley \textit{et al.}, 2005).

\textbf{1.2 Objectives}

This project pursues one main objective: to test several accepted electromagnetic (EM) theoretical equations that describe the relationship between surface GPR reflection amplitudes and sub-wavelength or ‘thin-layer’ fracture apertures. The experiments within this study are partitioned into two chapters that are designed to be submitted as stand-alone publications (hence there is some repetition in introductory material): Chapter 2, examining the relationship between the reflection amplitude and air-filled fractures; and Chapter 3, examining the relationship between the reflection amplitude of a water-filled fracture along with the impact of salinity and temperature on the reflection amplitudes. Several ancillary projects needed to be performed to clarify the main research goal, and are subsequently described in the Appendices.
2.0 Air-filled Fracture

2.1 Abstract

Several theoretical equations that predict sub-wavelength ‘thin-layer’ reflection amplitudes are compared to the results of a series of controlled ground penetrating radar (GPR) surveys using 1 GHz transducers over a physical model of a fracture. Two large plastic (UHMW-PE) blocks, separated by one or more stacked inserts (polyethylene; ~0.1 mm thick) for a total of 101 surveys, generate a modeled fracture from 0-300 mm in aperture. All of the theoretical reflection coefficient equations fail to predict observed reflection amplitude oscillations in the data when the fracture aperture is less than 1/48 of a wavelength ($\lambda$). The only theoretical formulation to properly predict any significant aspect of the fracture EM reflectivity is the Widess equation; however the best fit only occurs where aperture sizes are less than $\lambda/16$, not $\lambda/8$ as predicted. The frequency dependency of the Widess equation in conjunction with the oscillations suggests that small (sub-millimeter) apertures may not be resolvable if using GPR antennas with a center frequency less than 1 GHz.

2.2 Introduction

Fluid flow in the subsurface is an important research topic to relate to “clean” and “dirty” water-related issues. While fluid flow in an unconsolidated media is fairly well understood and predictable, fluid flow through a fractured media is poorly constrained and governing field-scale hydrologic equations are difficult to test. One method for simulating groundwater flow in fractured media is to model the target
environment as an equivalent porous media. Inherent in this model is the cubic law (Lamb, 1932) that demonstrates the significance of reliable fracture characterization, e.g., doubling the fracture aperture size will result in an 8-fold increase in discharge through a single fracture. Current methods for characterizing the properties involve questionable assumptions. For example, one method involves coring the media and measuring the properties from the core, which assumes the fracture properties remain constant under changing hydrostatic conditions (from the initial state to the core) and the core is representative of the subsurface at the field scale. Alternatively, modeling a fractured media involves analyzing hydraulic properties from monitoring wells and ignoring the fracture characterization altogether. Surface ground penetrating radar (GPR) surveys present an attractive alternative because the potential for 2.5D fracture distribution may be determined in situ.

2.2.1 Background

Electromagnetic wave theory that dictates operation of GPR is well understood (see Appendix 4; Baker et al., 2007) and through the 25+ year history of modern GPR significant and diverse applications hydrogeological investigations have been successfully carried out. One of the early applications included locating the water table in the subsurface (e.g., Sellman et al., 1983). Later applications have extended to locating non-aqueous phase liquids (NAPLs; e.g., Jordan et al., 2004), detecting and recognizing NAPLs both before and after degradation, (Redman et al., 1994 and Daniels et al., 1995, respectively), and locating large fractures or networks of fractures (e.g., Orlando 2003; Porsani et al. 2005; 2006).
Another recent investigation demonstrated the effectiveness of GPR in detecting sub-wavelength ‘thin-layer’ fractures with the addition of a saline tracer (e.g., Talley et al., 2005). In this study, the researchers used GPR to survey a fractured quartzose sandstone. To process the data and enhance the signal-to-noise ratio, they subtracted the data collected prior to the addition of a saline tracer from the data collected after the addition of a saline tracer and observed an amplitude anomaly in the subsurface. They confirmed the presence of a fracture measured at ~0.5 mm at a depth of 7 m with a borehole camera. Of principal importance to this study is the relationship between the fracture aperture and the GPR wavelength—a 3-order of magnitude difference—effectively demonstrating the practical application of detecting sub-wavelength ‘thin-layer’ fracture using GPR.

Reflection coefficients (R) for single source-receiver thick layers have been developed previously and can be found in many electromagnetic textbooks (e.g., Equation (1) and Table 1; Griffiths, 1999)). Modifications to this equation to account for source-receiver separation where the incident energy perpendicular to the target reflector in the transverse electric field (TE) are shown in Equations (2) and (3), with variables and identities defined in Table 1 (Annan 2005).

Determination of sub-wavelength reflection coefficient is complicated due to the interference between the top and bottom of the fracture. Attempts to quantify the relationship between the fracture aperture and reflected energy have been proposed by several authors (for EM: Hollender and Tillard, 1998; Annan et al., 1988;
Table 1 Common EM variables and identities.

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Variables in text:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\varepsilon_r)</td>
<td>Relative dielectric permittivity (unitless)</td>
</tr>
<tr>
<td>(\theta)</td>
<td>Angle of wave (degrees)</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>Wavelength (m)</td>
</tr>
<tr>
<td>(\nu)</td>
<td>Velocity (m/s)</td>
</tr>
<tr>
<td>(d)</td>
<td>Aperture size (m)</td>
</tr>
<tr>
<td>(f)</td>
<td>Frequency (Hertz)</td>
</tr>
<tr>
<td>(A)</td>
<td>Reflection amplitude for thick bed (mV)</td>
</tr>
<tr>
<td>(R)</td>
<td>Reflection coefficient</td>
</tr>
<tr>
<td>(T)</td>
<td>Transmission Coefficient</td>
</tr>
<tr>
<td>(x)</td>
<td>Number of reflections</td>
</tr>
</tbody>
</table>

Identities:

\[
R_{21} = -R_{12} \\
T_{12} = 1 + R_{12} \\
T_{21} = 1 - R_{12}
\]

*Subscript 1 and 2 represent the plastic and air media respectively, whereas \(i\) and \(t\) are the incident and transmitted wave angles.

\[
\frac{\varepsilon_1 - \varepsilon_2}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} = R \tag{1}
\]

\[
\frac{\cos \theta_1 \sqrt{\varepsilon_1} - \cos \theta_2 \sqrt{\varepsilon_2}}\cos \theta_1 \sqrt{\varepsilon_1} + \cos \theta_2 \sqrt{\varepsilon_2} = R_{TE} \tag{2}
\]

\[
\frac{2\cos \theta_1 \sqrt{\varepsilon_1}}\cos \theta_1 \sqrt{\varepsilon_1} + \cos \theta_2 \sqrt{\varepsilon_2} = 1 + R_{TE} = T_{TE} \tag{3}
\]
2004; and for seismology: Widess, 1973.). It should be noted, however, that in the literature, the term ‘thin-layer’ is often a source of ambiguity. For example, Widess (1973) defines it as a layer with a thickness of less than $\lambda/8$, Hollender and Tillard (1998) define it where the layer thickness is “small compared to wavelength,” And Annan (2005) defines a thin-layer as a vertical resolution limit with an equation (see Appendix 5). The objective of this paper is to examine preexisting theoretical equations and compare the expected results with experimentally determined thin-layer reflection coefficients for an air-filled fracture.

2.3 Methodology

2.3.1 Equipment

A Sensors & Software PulseEKKO Pro GPR unit with 1000 MHz shielded transducers (antennas) was used for data collection. Although lower-frequency antennas would have better represented the frequencies often used in the field, the high-frequency antennas allowed us to use smaller UHMW-PE blocks (though they still weighed over 345 kg). Data were collected using 0.1 ns sampling interval, 32 stacks (to enhance the signal-to-noise ratio), 25 ns total recording time, antenna separation of 0.15 m, antenna stepsize of 0.01 m, and a pulsar setting of 185,000 mV. The trigger was an odometer wheel set to trigger at the appropriate stepsize.

The site location of the study is within the University of Tennessee’s Plant Sciences experimental research station plot B4, located on the floodplain of the Tennessee River. This field location was selected to reduce the influence of ambient
noise associated with urban settings that could appear as real signal in our data (e.g.,
powerlines emitting an EM signal, surface clutter generating air-wave echo, etc.).
Prior to beginning the study, a GPR survey was performed at the test site location to
identify potential sources of noise such as buried objects, as the site has also been used
for hydrogeologic research and contains several wells (Figure 2.1). The survey aided
in the position of the experiment apparatus, as several metallic objects were detected
in the general region. The slightly uneven surface was leveled with the addition of
standard landscaping sand, and a 2.4 m x 2.4 m plywood box was built to protect the
experiment from the elements. This box was constructed entirely using plastic
fasteners or ‘zip ties.’

2.3.2 Material Properties and Model Design
The type of plastic chosen for the model is an ultra-high molecular weight
polyethylene (UHMW-PE). One advantage of the plastic blocks is the high volume
resistivity of $5 \times 10^{16}$ ohm-cm. This allows for the assumption that the block is a near-
lossless material with very low signal attenuation that aids in simplifying later
equations (e.g., Baker et al., 2007). The lateral dimensions of the two blocks used in
the experiment are roughly 1.25 m x 1.25 m (4 ft x 4ft), with the top block slightly
longer for lifting purposes. The thickness (measured using a caliper) of the top block
at the edges of the survey point varies by 0.3 mm (154.44 to 154.74 mm) and the
bottom block is approximately 101.6 mm thick (its thickness was only important
insofar as delaying reflections from the bottom of the box).
Figure 2.1 Blocks being lifted for the addition of inserts. The steel beam and hydraulic jacks were placed at a distance of no less than 4 meters during data collection. Plastic tarp and wooden box protect the blocks from the elements.
The block size was limited due to a combination of manufacturing constraints and the need to lift the top block. The top block has as much as 1.57 mm variation in thickness overall, but measured only 0.24 mm along the entire starting face (North side) to 0.7 mm along the entire end face (South side), with the majority of the end face having the variability of 0.03 mm. All surveys were taken down the center of the block to reduce the perpendicular edge noise.

Stacked inserts made of polyethylene measuring 0.098 ± 0.001 mm thick (herein approximated as 0.1 mm) were cut into 15-cm squares and placed between the blocks at each corner. The number of inserts at each corner was always the same; thus, the aperture of the simulated fracture was consistent for each data run. We ran a total of 101 surveys representing 101 different apertures. We used an increment of 1 insert (added to each corner) for the first 51 surveys (0 to 5 mm aperture at increments of 0.1 mm), then we added 2 inserts between each of the next 25 surveys (5.2 to 10.0 mm at 0.2-mm increments), followed by 5 inserts added between the next 20 surveys (10.5 to 20.0 mm at 0.5-mm increments). Additionally, several other surveys were taken with 300 inserts (3 cm), and 1 survey with 2.54 mm (1 inch) blocks of UHMW-PE in each corner; wooden posts were used for larger apertures of ~100, ~200, ~300 mm (4, 8, 12 inches respectively). This coarsening of sampling interval with increasing aperture size focused the attention on fractures which are more likely to exist in the subsurface (i.e., less than 1 cm). Data were collected over larger-aperture sizes to yield high-end constraints on the comparison to modeling equations.
and to examine the impact as the system shifts from a ‘thin-layer’ (subwavelength) to a ‘thick-layer’ (greater than a wavelength) model.

2.3.3 Sources of Error

Because the top block was supported in the corners by small inserts, it became necessary to calculate the potential beam deflection or ‘sag’ present. Beam deflection was estimated using a simple 2D equation (e.g., Gere, 2000) and yielded a maximum deflection of 0.05 mm at the center of the block (see Appendix 2). This calculation is based on a point-source support rather than a guided support over the entire 15-cm length of the inserts, resulting in a maximum estimate of deflection. The measured tolerance of the top block in the survey direction is 0.3 mm. This measured value is below the fabrication and planning tolerances (0.762 mm). As previously described, the site location was also carefully chosen to minimize error sources.

Preliminary surveys were taken orthogonal to the principle survey direction and compared. The results confirm the assumption of the plastic blocks being both homogeneous and isotropic at the scale of the experiment. This is important with respect to the survey because anisotropy could impact the averaged traces when the antenna cart shifted during data collection. The inserts measured with a digital caliper consistently at 0.1 mm for an individual sheet and 1.01 mm for 10 sheets, a difference of ~2% from the manufacturer’s suggestion. For the purpose of this research we will defer to the manufacturer’s measurements as our caliper accuracy was not as great.
Dielectric permittivity of the plastic blocks was determined using a Stevens’ Hydra Probe that has an accuracy of 0.2 (unitless). The average of 1104 tests on a single sample is 2.05, with a range from 1.99 to 2.09. The error associated with the probe is ± 0.2, but for the purpose of this project we will assume a dielectric permittivity of 2.0 for ease of calculations. The use of the probe allows for an independent determination of the dielectric constant.

The velocity of the EM wave in the plastic media is governed by the dielectric permittivity of the plastic media (see Baker et al., 2007). As such, we calculate the EM propagation velocity of the UHMW-PE blocks as 0.2 m/ns.

2.3.4 Parameters

Angle of incidence (θi = 26°) is calculated using the known geometry of the blocks and the antennae separation (Figure 2.2). Velocity (0.2m/ns), wavelength (0.2m), transmitted angle (θt = 39°) are calculated using basic principles as outlined in previous publications (e.g., Baker et. al, 2007). For the single source-receiver approximation we used the vertical thickness (D) of the top block and to account for the dual antennas we used the ray path (H) for our calculations (Figure 2.2).
Figure 2.2 Simplified reflection and transmission ray paths for dual antennas.
2.3.5 Processing

The overall processing goal was to preserve reflection amplitude values. The data were processed starting with Sensors and Software’s EKKO View Deluxe. The horizontal positions were cropped to include only the middle 41 traces to reduce the influence of the edge effects that are apparent in the profiles (Figure 2.3). The positions were cropped further to obtain the largest set of consistent readings. The time zero setting was set for 1% of the maximum amplitude, to remove the prepulse data. The time window was then cropped for a new time-duration window of 10 ns to focus on the system and not the energy as it transmitted into the subsurface. A DEWOW filter was then applied to remove very-low-frequency noise. The profiles are averaged to give a single trace per incremental change, and interpolated to 0.05 ns sampling interval (the last few profiles were collected with a 0.05 ns sampling interval). The interpolation was done to reduce the chance of processing artifacts. The data were exported to Parallel Geosciences’ Seismic Processing Workstations (SPW) v2.2.7. A static shift then applied to the traces to match up the air wave peak amplitudes. This was performed in 3 iterations to minimize cycle skipping.

The direct wave value increased with increasing aperture in our surveys. This suggests there is some interference on the direct wave from the fracture aperture as we assume the direct wave of the plastic block should not change. To remove this feature and instrument variability, the data were normalized by the amplitude of the direct wave at any aperture against the direct wave value with no inserts (N₀). The normalizing factor was then multiplied by the difference between the Nth trace and the N₀ peak amplitude, to obtain a normalized reflection amplitude. The N₀ trace was
then subtracted from itself and every subsequent trace to show the influence of the change in aperture without any interference from the direct wave or air-coupled wave (Figure 2.4). To examine the impact of the normalization on the reflection amplitude values we calculated the maximum reflection amplitude from Equation (8) (below) at 84,300 mV. The normalized maximum reflection amplitude (evaluated when wavelength = aperture) is 81,300 mV, a difference of 4%. This step is interpreted as enhancing the signal to noise ratio.

2.3.6 Prior Reflection Coefficient Studies

Several theoretical reflection coefficients and a modified form will be examined for comparison with the normalized observed data (herein referred to as simply “observed” for brevity). The details of the derivation of each equation are available in the original publications or explained below, and all variables are defined in Table 1. Equations (4, 5, 6) are EM based equations which account for the interference between the top and bottom reflections superimposed, effectively treating the thin-layer as a boundary.

Hollender and Tillard 1998

Equation (4) accounts for the source-receiver separation by using the $R_{TE}$ and $T_{TE}$ components described in Equations (2) and (3) for obtaining the single interface reflection and transmission coefficient.

$$R_{HT} = R_{12} + T_{12}T_{21} \sum_{x=1}^{\infty} R_{21}^{(2x-1)} e^{-\frac{4\pi}{\lambda_2} \cos \theta_2}$$  \hspace{1cm} (4)
Figure 2.3 Steps through the processing using EKKO Viewer Deluxe. A is the raw data with the edge effects resulting in a ‘x’ pattern in the data. B is the same data with a Time Zero and Time Window change. After a position crop and DEWOWing, C, the data averaged to a single trace, D, before exporting to SPW. Red Arrows indicate fracture aperture location.
Figure 2.4 Averaged trace for incrementally increasing air-filled aperture. Left: The airwave is observed at 0 ns (due to processing), the direct wave at 1.2 ns, and the fracture reflection beginning at ~1.6 ns is not clearly defined. Right: Subtraction of the 0th trace enhances the signal due to the fracture aperture change (red arrows).
Annan 2005

A simplified solution for solving reflection coefficients valid for a single source-receiver but often Equation (5) uses the R form for obtaining the reflection and transmission coefficients, and the identities listed above. This single antenna approximation used in this equation is common among GPR users.

\[ R_A = R_{12} + \frac{T_{12}T_{21}R_{21}e^{j\frac{\pi\theta_2}{2}}}{1 - R_{21}^2 e^{j\frac{\pi\theta_1}{2}}} \]  

(5)

Modified Annan 2005

Equation (6) is a modified version of Equation (5) accounting for source-receiver offset via Equations (2) and (3).

\[ R_{AM} = R_{12} + \frac{T_{12}T_{21}R_{21}e^{j\frac{\pi\theta_2}{2}}\cos\theta_2}{1 - R_{21}^2 e^{j\frac{\pi\theta_1}{2}}\cos\theta_2} \]  

(6)

Widess 1973

In a classic seismology paper discussing reflection coefficients for a thin-layer, Widess (1973), a thin-layer is defined as a resolution limit where aperture is \(< \lambda /8\).

The reflection amplitude for a very thick bed (A) is determined using Equation (8) (Annan 2005) and used to calibrate the Widess equation for the maximum amplitude.

This equation is fundamentally different from the EM equations as it does not account for the source-receiver geometry but is concerned primarily with the interference between the top and bottom reflections.
\[
R_w = \frac{4\pi 4d}{\lambda^2}
\]  
(7)

\[
R_{\text{Max}} = R_{12} \left( \frac{2}{1 + R_{12}^2} \right)
\]  
(8)

2.4 Results

The observed reflected amplitude (Figure 2.5A) in the full range approaches a maximum value as the aperture approaches \(\lambda/3\). But this may be an artifact of sampling as no data was collected between \(\lambda/6\) to \(\lambda/3\). The data oscillates throughout the range of apertures, and is especially prominent at the fine scale (Figure 2.5: B, C, and D). The amplitude of the oscillation decreases with increasing aperture. This secondary oscillation was not predicted by any of the tested equations and the possible sources for which are discussed below.

Annan (2005) and Modified Annan

Both the single source-receiver equation and the modified form follow a similar trend. As expected the single source-receiver equation underestimated the general trend of the observed data more than the modified form. Both forms contain a primary oscillation as demonstrated by Annan (2005) as \(\lambda/\text{aperture}\) increased. For the range of aperture sizes in this project, only 1 primary oscillation is observed, where \(\lambda = \text{aperture}\) as we did not have a data point at \(\lambda/2\), where the first primary oscillation should be seen. Our observed data did not have the large scale oscillation which is present in both equations and neither equation matched the observed data.
Hollender and Tillard (1998)

This boundary equation did not follow the observed pattern but did match the observed reflection amplitude when the aperture is equal to $\lambda$ (results not shown) and showed a relatively good fit where aperture equals $\lambda/3$ (100 mm); this is no longer ‘thin-layer’ and not likely representative of a real subsurface fracture. The primary oscillation predicted by this equation was not present in the observed data.

Widess (1973)

This equation when scaled using Annan’s (2005) maximum value reflection coefficient, best fitted the observed trend for aperture sizes below $\lambda/16$. Widess (1973) defined the applicability of this equation where apertures were below $\lambda/8$; however, our results show that the equation is only valid below $\lambda/16$ where the predicted values match a trendline through the observed data (Figure 2.6) with an error of 4%.

2.5 Conclusions

No equation predicted the secondary reflection amplitude oscillations prominent at apertures less than $\lambda/48$ (~6 mm; Figure 2.5). The source and accurate prediction of these oscillations must be resolved before any reasonable determination of aperture size from GPR reflection amplitude can be resolved for apertures less than $\lambda/48$. For example, in our data a 10,000 mV reflection amplitude could fit numerous
apertures between 0.5 and 4 mm, resulting in a 512-fold increase in discharge given the cubic relationship between discharge and aperture size (Lamb, 1932). These small scale oscillations dampen out at larger apertures, resulting in an increased accuracy of aperture predictions where aperture size is greater than $\lambda/10$ (>10 mm).

Three of the equations—Annan, 2005; Modified Annan; Hollender and Tillard; 1998—predict a primary oscillation at large apertures that is not seen in the observed reflection amplitudes. The plot of the three EM equations start out concave for apertures <20 mm but overall has a convex pattern at larger apertures. Both Annan equations (2005 and modified) underestimate the value at every aperture. The Hollender and Tillard (1998) varies little for apertures <20 mm, then overestimates the observed reflection amplitude, oscillates, and matches the observed reflection amplitude only where $\lambda$ equals aperture. At aperture sizes below $\lambda/16$, the three EM equations are very similar to one another.

The Widess (1973) reflection amplitude is sufficient for apertures less than $\lambda/16$ (19 mm) for our GPR data, whereas the author predicted the equation would be valid for apertures less than $\lambda/8$ for seismic data. The best fit trendline for the observed reflection amplitude is within 4% of the Widess (1973) equation. The frequency dependency of this equation suggests that the oscillations will dominate the data at frequencies below 1 GHz; thus, field predictions of aperture size using GPR may be highly inaccurate for lower-frequency GPR antennas (e.g., 100 MHz).
Figure 2.5 Predicted and actual reflection amplitudes with aperture variation for an air-filled fracture. A is the full range of data and B, C, and D are smaller scaled which show the oscillation in the observed data.
Figure 2.6 Trendline fit comparison between Widess (1973) and observed reflection amplitudes for thin layers.
Possible sources of the secondary oscillation in the observed data include: 1) detection of the roughness or small undulations within the thickness of the aperture due to the blocks not being perfectly planed surfaces, 2) resulting from the sampling interval being large compared to the aperture size, 3) resulting from some real component of the signal not yet understood.

2.6 References


Widess, M.B., 1973, How thin is a thin bed?: Geophysics, 38, No. 6, p. 1176-1180.
3.0 Water-filled Fracture

3.1 Abstract

Current methods of collecting data for modeling groundwater flow in a fractured media (e.g. fractured bedrock) involve expensive and invasive procedures that typically yield poorly-constrained results due to highly spatially variable fracture apertures and the resulting channelization. Surface ground penetrating radar (GPR) surveys present an attractive alternative because of the full two-dimensional distribution of fracture aperture may be determined. Typical fractures have sub-wavelength apertures (i.e. are considered “thin-layers”) and fluid flow through fractures is governed by the cubic law; therefore, precise aperture characterizations are critical. The focus of this study was to examine the theoretical EM equations for fracture characterization with respect to 1) thin-layer vertical resolution of water-filled fractures, 2) thin-layer vertical resolution of a fluid-filled fracture with variable conductivity (salinity). The physical model included 2 UHMW-PE (ultra-high molecular weight polyethylene) blocks that have EM properties of real earth materials, separated by thin (0.1 mm) polyethylene inserts to create a range of apertures for the first two phases. Air bubbles trapped within the small apertures resulted in a multi-phase and variable signal at the fracture, so this portion was abandoned. For a water-filled constant aperture with variable conductivity (0-5700 mS/m) the increase in salinity resulted in a decrease in reflection amplitude.
3.2 Introduction

In situ fracture characterization remains an ideal yet difficult goal for modeling groundwater flow in a fractured media. The current methods are both invasive and expensive and typically yield poorly constrained results due to the high spatial variability of fracture apertures. Surface ground penetrating radar surveys (GPR) present an attractive alternative because of the full 2D and pseudo-3D distribution of fractures may be characterized.

In the first phase of this project (Burns and Baker, 2008) researchers used an idealized fracture model to collect reflection amplitudes for 101 different aperture sizes. The tested theoretical EM equations failed to match the observed reflection amplitudes for a thin-layer. A thin-layer approximation originally developed for seismic reflection (Widess 1973) did fit the data reasonably well (Figure 3.1; 4% difference in coefficients) but for apertures smaller than predicted by Widess (<\lambda/16 versus <\lambda/8). Additional details on the fundamental differences in the equations can be found in the preceding Chapter.

The detailed physics of GPR function can be found in many publications (e.g., Baker et. al, 2007) and will only be very briefly described here (additionally see Appendix 4). GPR function is analogous to seismic wave propagation. As energy moving through the subsurface reflects with changes in bulk modulus in seismic reflection, in GPR, energy reflections are governed by changes in the dielectric constant from one medium to another.
Previous studies (e.g. Topp et al., 1980, Davis and Annan, 1989) have demonstrated changes in salinity (Cl⁻) do not impact the change in real component of the dielectric constant but do alter the imaginary component of the dielectric constant, which is manifested as an attenuation of the signal. The inference is the initial reflection from the top of a layer of any thickness would not be impacted by changes in conductivity (salinity) in the second layer (fracture aperture). However, as the energy propagates through the fluid-filled layer, changes in conductivity will attenuate the signal and could effectively result in no data from the second layer. In terms of a thin-layer, the reflection amplitude values increase with increasing aperture size due to interference. If the signal from the second layer is attenuated, then the resulting reflection amplitude should only be from the top layer; this would result in reflection amplitude for a thin-layer equal to the reflection amplitude for a thick layer. Recent field research (Talley et al., 2005) concluded in a qualitative study, the addition of a saline tracer to a fluid-filled fractured sandstone did impact reflection amplitudes, determined via a background subtraction each survey without the tracer.

For the second phase of this project the objectives are to 1) re-examine the theoretical equations for predicting ‘thin-layer’ reflection amplitude for a fluid-filled saturated fracture and 2) examine the influence of salinity on the reflection amplitude at a constant aperture.
Figure 3.1 Trendline fit comparison between Widess (1973) and observed reflection amplitudes for thin layers (Burns and Baker 2008)
3.3 Methodology

The site location, equipment used, processing steps, and the physical model of an idealized fracture are fully explained in the first phase of this project (see Chapter 2). A brief description of the model is explained below in addition to the changes for the new experiments.

3.3.1 Base Model

The model consists of two 1.2 m x 1.2 m (4 ft. x 4 ft) UHMW-PE blocks (Figure 3.2). The top block is 0.154 m (6 in) thick and the bottom block is ~0.1 m (4 in) thick. The top block is slightly longer than the bottom block for lifting purposes. The blocks were located within a pool, and inside of a wooden box. With the addition of a plastic tarp, the wooden case provided protection from the elements.

3.3.2 Equipment

A Sensors and Software Pulse EKKOPro GPR unit with 1GHz antennas, with 32 stacks, 0.05 ns sampling interval, an odometer trigger, 185v pulsar setting, and 0.15 m antenna separation was used for reflection amplitude data collection. Time, temperature, total dissolved solids, specific conductivity, conductivity (temperature corrected) and salinity (temperature corrected; calculated from conductivity) was collected using a handheld YSI Multimeter Model #556 calibrated with YSI standards.
Figure 3.2 System set up with water filled fracture.
3.3.3 Experimental Design

3.3.3.1 Constant conductivity, variable aperture
Using the base model described in the preceding chapter, distilled water was added to the edges of the pool up to 18 cm (~7 in). No initial precautions were considered for air bubbles entering the fracture. This shallow water depth was necessary to keep the blocks from floating (density of UHMW-PE is 0.93 g/cm³) but sufficiently deep to saturate the fracture. Polyethylene inserts (15cm x 15cm x 0.01 cm) were placed into the corners of the lower block in increments of 2, 5, 10, 15, 20, and 50. Surveys were taken down the center of the block to mitigate edge effects.

3.3.3.2 Variable conductivity, constant aperture
Also using the base model, surveys were run with a constant aperture of ~0.5 mm (50 inserts). Table salt (halite) was added evenly around the blocks inside the pool and mixed by hand, and/or with an electric pump and allowed to equilibrate for no less than 20 minutes (and monitored at 2 points per each side around the blocks) with the multimeter in incremental concentrations of 0, 500, 600, 700, 800, 1000, 1100, 1300, 1500, 1700, 2000, 2400, 2900, 3600, 4000, 4800, and 5700 mS/m. Surveys were taken down the medial axis of the block to mitigate edge effects.

3.4.3 Processing
All of the water-filled fracture surveys were minimally processed (see Chapter 2) to reduce the influence of edge effects but minimize processing artifacts. A low-frequency time-filter (DEWOW) was applied to reduce the influence of signal decay
and the remaining traces \((20 < x < 42)\) were averaged to obtain 1 trace for each 
aperture, temperature, or salinity increment. All traces were subject to a static 
adjustment, but no time zero correction was applied because depth was already 
known.

### 3.4 Results

#### 3.4.1 Constant salinity, variable aperture

The preliminary surveys included aperture sizes of 0, 0.2 mm, 0.5 mm, 1.0 mm, 
1.5 mm, 2.0 mm, and 5.0 mm filled with distilled water. Upon examination of the 
data, there was some variability in the signal attributes at the time associated with 
the fracture (~2.5 ns, two-way travel time, Figure 3.4). This variability was present 
in some of the surveys, but not in the larger 5.0 mm aperture size (Figure 3.5). We 
interpreted this to be the result of the possible presence of small air bubbles trapped in 
the fracture aperture due to some air bubbles escaping when we lifted the top block 
for larger apertures. Uneven lifting of the top blocks, pumping water into the fracture 
aperture with electric pumps, and ‘pumping’ by raising the top block and pushing 
down on the bottom block did not consistently manage to remove this variability. As 
a result, this portion of the project was abandoned.
Figure 3.4 Survey of a water-filled 1.5 mm aperture. Note at approximately 2.5 ns (yellow arrows), the location of the fracture, the signal attributes are highly variable throughout the traces.
Figure 3.5 Survey of a water-filled 5 mm aperture. Yellow arrows indicate consistency in the signal in response throughout the traces.
3.4.2 Variable conductivity, constant aperture

Large contrasts in dielectric constant will result in large reflection amplitudes. In the previous chapter, the difference between the (real component) dielectric constant of the materials was 1, for these surveys the difference is ~78. A first order approximation (Equation (1), Chapter 2) was used to predict very large reflection amplitude for a water-filled fracture. This was confirmed in the data (Figure 3.6). Multiples are commonly associated with large or ‘bombing’ reflections and this was also present in the data (Figure 3.6). Unfortunately, the GPR data logger did not record (or ‘clipped’) the large reflection amplitudes for all of the surveys. The presence of a multiple from the water-filled fracture allowed for continued analysis (Figures 3.6 and 3.7). While these values do not represent the actual reflection amplitude at the interface, they do represent a scaled down version resulting from additional reflections within the top plastic block. It is important to note the plastic block is a lossless material, and the scaling factor is assumed to be constant, and without attenuation for the reflection at the top interface.

In 4 of the 17 surveys, the reflection amplitudes were greater than 5% deviation from the mean value. This is consistent with instrument variability observed throughout our surveys (Appendix 2). However, there is a trend data where the reflection amplitudes increase with increasing conductivity (Figure 3.8, 3.9) and we obtain an $R^2$ value of 0.53. Furthermore, the trend appears to be segmented, that is from 0-2000 mS/m, the reflection amplitude is decreasing with increasing conductivity, and from 2400-5700 mS/m, the reflection amplitude is increasing with increasing
conductivity. In lieu of the small signal variability above background, the relationship between reflection amplitude and conductivity is still statistically significant, that is, there is a positive correlation between reflection amplitude and conductivity increase. Additionally, the attenuation associated with the increase in conductivity is observed in reflections which have passed through the water-filled fracture (Figure 3.7).

3.6 Conclusions

Our experimental design or model was not sufficient to survey very small water-filled fractures ($<\lambda/7$) due to air bubbles trapped between the two blocks. The surveys also failed to successfully image the reflection amplitude at the fracture for a constant fracture aperture of 5 mm because of large amplitude ‘clipping’ by the data logger of the GPR unit. As a result the fracture multiple was used for the analysis. The results from the fracture multiple analysis demonstrate a decrease in amplitude with an increase in conductivity (salinity). This relationship indicates the reflection amplitude contribution from the bottom fracture reflector is lost due to increased signal attenuation (resulting from the increased conductivity).

Prior field research which motivated this project (Talley et al., 2005) used a saline tracer and GPR to locate a 0.5 mm size fracture in the subsurface with 100 MHz antennas. In their survey, the researchers detected an increase in reflection amplitude with increasing conductivity (salinity).
Figure 3.6 Average trace for each conductivity survey at a constant aperture. Traces are increasing in salinity from left (0 mS/m) to right (5700 mS/m). Polarity is reversed in this image for ease of displaying.
Figure 3.7 Superimposed traces corresponding to conductivity change. Yellow arrow indicates the ‘clipped’ reflection amplitude at the fracture, red arrow indicated the fracture reflection multiple, and blue arrow points to a reflection attenuating with increased salinity. All files were static shifted to the clipped fracture reflection. At the blue arrow, the top (light gray) trace corresponds to the distilled water and the bottom trace (black) corresponds to saltwater.
Figure 3.8 Reflection Amplitude with conductivity. Two distinct trends are observed from 0-2000 mS/m and 2400-5700 mS/m.
In spite of the processing difference (e.g. they applied an envelope that changed the sign of the amplitude) there was still the discrepancy between the survey results. This may be explained by a difference in relative aperture size. In their survey, the aperture was \( \lambda/670 \), whereas for this project the aperture size was \( \lambda/6.7 \). From Chapter 2, the sampling interval was focused primarily on very thin layers that resulted in destructive interference between the top and bottom layers. Though not observed, it is possible that the reflections from the top and bottom interface could be constructive for larger aperture sizes. In this case, the increase in conductivity would result in a decrease in contribution from the bottom fracture reflector and an overall decrease in reflection amplitude.

The implication of these phenomena is that the addition of a high concentration saline tracer will remove the bottom fracture reflection and the total reflection amplitude should be equivalent to that of a thick-layer. By adding a high concentration saline tracer for very thin apertures (destructive interference) the resulting signal will increase in amplitude aiding in fracture detection. Also, a qualitative characterization can be performed on the fracture based on increase or decrease in the overall reflection amplitude with increasing salinity.
3.7 References


Burns, K.E., G.S. Baker, submitted, Comparing theoretical equations to empirical results of the relationship between surface ground penetrating radar reflection amplitudes and horizontal sub-wavelength (thin-layer) fracture apertures: Geophysics.


Widess, M.B., 1973, How thin is a thin bed?: Geophysics, 38, No. 6, p. 1176-1180.
4.0 Conclusions

Several of the commonly used equations for describing EM reflections, resolution limits, and antenna illumination have failed to accurately predict known values of an idealized fractured media at 1 GHz. The Widess (1973) seismic equation did reasonably match this project's observed data, but at half of the predicted aperture size (λ/16). None of the equations examined predicted the secondary oscillation observed in the air-filled sub-wavelength aperture portion of this study (Chapter 2). Additional experiments were performed to examine possible sources of error, including instrument response to temperature fluctuations and thermal expansion of the inserts (Appendix 2). The thermal expansion of the inserts was concluded to be negligible, and the temperature fluctuation study indicated a negative correlation between reflection amplitude and increased temperatures, opposite of the observed air-filled fracture data. This oscillation must be resolved to apply the predictive application of the Widess equation. The importance of this can be observed when trying to fit a 10,000 mV reflection amplitude to our observed data (0.5 – 4 mm), this large range can result in up to a 512-fold increase in discharge using common hydrogeologic equations (Snow 1968, Snow 1969).

The second portion of this project focuses on the influence of salinity on thin-layer GPR reflection amplitudes (Chapter 3). The changes in salinity from 0 to 5700 mS/m (distilled to saltwater) at a constant aperture resulted in decreased contribution from the bottom fracture reflector (due to signal attenuation). Comparisons between
the non-conductive (distilled water-filled fracture) and the conductive (highly saline water) may allow the operator to qualitatively determine a coarse aperture size for a fracture based on the constructive or destructive relationship of the non-conductive water-filled fracture.
LIST OF REFERENCES
References


Widess, M.B., 1973, How thin is a thin bed?: Geophysics, 38, No. 6, p. 1176-1180.
APPENDICES
Appendix 1: Noise and Cable Study

Examining the influence of noise associated with cable placement and cable movement is an important for quantified GPR research. The manufacturer suggests coaxial cables will not generate noise when used only in conjunction with shielded antennas, as in this thesis.

To test their claim, a new survey design was developed. The antennas were placed in the center and on top of the plastic blocks with a one second sampling interval without moving the antennas. At the start of the survey, the operator along with the battery and data logger, was extended the length of the cables, 10 m. After ~10 seconds the operator then walked with the battery and data logger toward the antenna, ~20 seconds, then setting down the data logger and battery and waiting out the remainder of the first minute (red shaded region, Figure A1). The operator then coiled (~15 seconds) and placed the coaxial cables next to the active antennas and collected data for remaining minute (green shaded region of Figure A1). This portion was designed to test the influence of the cable position and the sensitivity of the data logger/battery position. To examine the potential influence of the movement of the cable as a possible source for signal noise, the operator tapped on the receiving antennae for ~30 seconds (yellow shaded region, Figure A1), then tapped on the transmitting antenna for ~30 seconds (orange shaded region, Figure A1) then tapped on both antenna for ~30 seconds (blue shaded region, Figure A1), then collected ~60 seconds worth of data with no movement (non-shaded region, Figure A1).
Results

There was little to no noise when tapping on the wires. The movement of the data logger, operator, and battery provided some discrete noise events associated with the battery and data logger being placed inside of the box. The coiling of the wires induced some noise, possibly associated with antennae movement. The placement of the wires next to the antennas on the block did not generate detectable noise. The type of noise observed in the profile (Figure A1) is primarily associated with a shift in the time and not a change in amplitude which can be observed by scrolling through the individual traces. Amplitude values for an arbitrarily chosen reflector at 4.3 ns (Figure A1) are plotted categorically in Table A1 below. The results indicate an overall increase in amplitude at each stage of testing.

Table A1

<table>
<thead>
<tr>
<th>Test</th>
<th>Mean Amplitude (mV)</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walking</td>
<td>25170</td>
<td>1080</td>
</tr>
<tr>
<td>Coiling Wires</td>
<td>25209</td>
<td>1205</td>
</tr>
<tr>
<td>Tapping Rx Cable</td>
<td>25464</td>
<td>869</td>
</tr>
<tr>
<td>Tapping Tx Cable</td>
<td>25680</td>
<td>699</td>
</tr>
<tr>
<td>Tapping Rx and Tx Cables</td>
<td>25768</td>
<td>1267</td>
</tr>
<tr>
<td>Rest</td>
<td>25863</td>
<td>857</td>
</tr>
<tr>
<td>TOTAL</td>
<td>25495</td>
<td>1059</td>
</tr>
</tbody>
</table>
Conclusions

The noise observed by the different tests did not impact the amplitude values above background variability. There was some time-shift associated noise with movement of the antennas, which can easily be corrected with additional software processing. The overall increase in mean amplitude values as we change from one stage of testing to another is possibly due to the change in temperature in conjunction with normal instrument variability.
Figure A1 Noise and Cable Profile. (Top) Red section indicates movement of operator, battery and data logger, green section indicates coiling of cables and placement next to the antennas, yellow, orange, and blue sections represent tapping on the receiving, transmitting, and both cables respectively. The non-shaded portion is at rest. (Bottom) A plot of the amplitude values.
Appendix 2: Error Sources

Part 1: Thermal Expansion

In Chapter 2 of this thesis, an oscillation in the air-filled thin-layer fracture, which were unpredicted by the tested theoretical equations was observed. Collaborators suggested that air temperature fluctuations could be a source of the oscillation observed. To determine if the oscillation was a physical response to temperature fluctuations, the thermal expansion of the inserts was tested using a representative sample of 10 inserts at various temperatures using a set of calipers for measuring. The inserts were first placed the 10 inserts in a -1°C freezer for 36 hours, and then placed the sample in an oven at 90°C for 2 hours, and finally allowed the sample to approach room temperature of 16°C.

The results for the room temperature and freezing were indistinguishable, which ranged from 1.01-1.02 mm. At 90°C the thickness ranged from 1.02-1.03 mm. This large temperature range overestimated the field conditions in the air-filled fracture phase of this project and the response of the inserts to these temperature ranges cannot account for the oscillations observed in the field data.

Part 2: Beam Deflection

The deflection or ‘sag’ in the top block when it was supported only by the 15.2 cm (6 inch) inserts in the corners was calculated. The equations (11, 12) procedures for 2D calculations can be readily found in most engineering text books (e.g. Gere 2000).
\[ Y_{\text{max}} = -\frac{0.0197 \times W \times L^3}{E \times I} \]  
\[ I = \frac{B \times H^3}{12} \]

<table>
<thead>
<tr>
<th>Meaning</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load</td>
<td>( W )</td>
<td>2066 N</td>
</tr>
<tr>
<td>Length of Beam</td>
<td>( L )</td>
<td>86 cm</td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>( E )</td>
<td>6,900,000 N/cm</td>
</tr>
<tr>
<td>Moment of Inertia</td>
<td>( I )</td>
<td>750 cm(^4)</td>
</tr>
<tr>
<td>Width of beam</td>
<td>( B )</td>
<td>2.54 cm</td>
</tr>
<tr>
<td>Height of beam</td>
<td>( H )</td>
<td>15.2 cm</td>
</tr>
<tr>
<td>Y-Max</td>
<td>Maximum Deflection at the Center of the Beam</td>
<td>-0.005 cm</td>
</tr>
</tbody>
</table>

It is important to note that these equations are based on point-support and universal loading. For the length of the beam, the length of the inserts (15.2 cm each) was subtracted from the total length of the block (recall the block is 123 cm), but the bottom block is 117 cm). The total weight of the top block was maintained in the calculations. This results in an overestimation in the maximum deflection, which is approximately \( \frac{1}{2} \) the thickness of 1 insert (0.1 mm) or 0.05 mm. This maximum deflection would likely be reduced further for the water-filled fracture study.
Part 3 Air-temperature

Hourly air temperature data from a nearby (<9.5 km) meteorological station at the McGee-Tyson Airport was obtained for the air-filled survey (Chapter 2). The preliminary analysis (Figure A2) suggested that the overall pattern displays an increase in reflection amplitude values associated with changes in air temperature. Using the base model (Chapter 3), a survey was designed to test the air-temperature influence with a constant water-filled fracture aperture at a constant salinity (600 mS/m)) for approximately 6 hours by running repeated surveys throughout the day. A multimeter with a sampling interval of 1.0 s was used to record the air temperatures for the full 6 hours. Air temperatures were averaged over the full minute in which the survey was taken. As with the preceding designs, all surveys were taken down the center of the block to mitigate edge effects.

Because of the clipping by the data logger of very large amplitudes values (described in Chapter 3), no analysis of the reflection amplitude at the fracture was possible. The multiple fracture reflection was used for the analysis (Figure A3). The multiple reflection amplitude values were mostly within 4% of the mean value; 3 of the 30 traces were between 5-6% above the mean.

Communications with a representative of Sensors and Software (the manufacturer of the GPR unit) suggested based on their experiments, the air temperature fluctuations would have little to no influence on the data (within 1-2%). In these surveys, the instrument variability was greater. The overall trend suggests a negative correlation
between the reflection amplitude with increasing temperature. This is inconsistent with
the air-filled fracture aperture data and we believe cannot be a source for the oscillation.

References:

Figure A2 Preliminary results on temperature as it relates to aperture size and reflection amplitudes.
Figure A3 Plot of the absolute value of the reflection amplitude with temperature change.
Appendix 3: Typical EM Properties of Earth Materials

Table A3: Electrical Properties of Geological Media

<table>
<thead>
<tr>
<th>Material</th>
<th>Dielectric constant</th>
<th>Conductivity (mS/m)</th>
<th>Velocity (m/ns)</th>
<th>Attenuation (dB/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>1</td>
<td>0</td>
<td>0.3</td>
<td>0</td>
</tr>
<tr>
<td>Distilled water</td>
<td>80</td>
<td>0.01</td>
<td>0.033</td>
<td>0.002</td>
</tr>
<tr>
<td>Fresh water</td>
<td>80</td>
<td>0.5</td>
<td>0.033</td>
<td>0.1</td>
</tr>
<tr>
<td>Sea water</td>
<td>80</td>
<td>3,000</td>
<td>0.01</td>
<td>103</td>
</tr>
<tr>
<td>Dry sand</td>
<td>3-5</td>
<td>0.01</td>
<td>0.15</td>
<td>0.01</td>
</tr>
<tr>
<td>Saturated sand</td>
<td>20-30</td>
<td>1-10</td>
<td>0.06</td>
<td>0.03-0.3</td>
</tr>
<tr>
<td>Limestone</td>
<td>4-8</td>
<td>0.5-2</td>
<td>0.12</td>
<td>0.4-1</td>
</tr>
<tr>
<td>Shale</td>
<td>5-15</td>
<td>1-100</td>
<td>0.09</td>
<td>1-100</td>
</tr>
<tr>
<td>Clay</td>
<td>5-40</td>
<td>2-1,000</td>
<td>0.06</td>
<td>1-300</td>
</tr>
<tr>
<td>Granite</td>
<td>4-6</td>
<td>0.01-1</td>
<td>0.13</td>
<td>0.01-1</td>
</tr>
<tr>
<td>Salt (dry)</td>
<td>5-6</td>
<td>0.01-1</td>
<td>0.13</td>
<td>0.01-1</td>
</tr>
<tr>
<td>Ice</td>
<td>3-4</td>
<td>0.01</td>
<td>0.16</td>
<td>0.01</td>
</tr>
<tr>
<td>UHMW-PE†</td>
<td>2.0-2.3</td>
<td>0</td>
<td>0.20</td>
<td>0</td>
</tr>
</tbody>
</table>

Electrical properties of geological media [Modified after Annan., 2005]; † = This study

References:

Appendix 4: How GPR Works

A high frequency (25 MHz – 1000 MHz) EM pulse is transmitted from a radar antenna into the ground for 10s to 100s of nanoseconds. A portion of the radar pulse is reflected at various interfaces and returned to the surface where a second receiving antenna collects the data. Whereas in seismic surveying, the reflections are governed by changes in the bulk and shear moduli of different layers, in GPR the reflections are governed by changes in EM properties, specifically the dielectric permittivity and magnetic susceptibility, which also corresponds to a change in the velocity of the pulse through the media. For most earth materials of interest, the magnetic susceptibility is approximated as unity (See Appendix 3 for more EM properties of real earth materials).

The most common survey designs are the common offset and common midpoint (Figure A4). The common midpoint (CMP) is where the operator increases the distance between the transmitting and receiving antennas by a constant increment. As the antennas are separated by a known distance, the slope of the reflected line at that interface can be measured to determine the velocity of the pulse through the medium. Common offset (CO) acquisition mode is where the operator moves the transmitting and receiving antennas at a common distance across the surface. In this mode, the operator is imaging a 2D profile of the subsurface.
Fig. A4: Common offset and common midpoint survey modes. (Annan, 1992)

References:

Appendix 5: Thin-layer Defined

Annan (2005) defines a resolution limit for distinguishing between two targets with a GPR unit with Equations (13, 14) below. This was interpreted as the definition of a thin-layer. The value obtained by solving these equations for our parameters, matched the change in curve (Figure 3.4A) where the reflection amplitude values flatten out (50 mm).

\[ \Delta r = \frac{Wv}{4} \]  \hspace{1cm} (13)

\[ W = \frac{1}{f_c} \]  \hspace{1cm} (14)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definitions</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>Pulse width (s)</td>
</tr>
<tr>
<td>v</td>
<td>Velocity (m/s)</td>
</tr>
<tr>
<td>r</td>
<td>Resolution Limit (m)</td>
</tr>
<tr>
<td>fc</td>
<td>Center frequency (1/s)</td>
</tr>
</tbody>
</table>

References:

Appendix 6: Antenna Illumination and Horizontal Resolution

The large discrepancy between the theoretical reflection amplitude equations for a vertical thin-layer (that is essentially a component of vertical resolution) warranted a test of the horizontal resolution theoretical equations applied to a vertically thin-layer (Figure A5).

The first step was to determine the size of the illumination ellipse (Figure A6). Equation (10) predicted that the illumination ellipse for our given model would be 23 cm x 11.5 cm (9 in x 4.4 in) corrected for the antenna separation. Due to the antennas broadfire array, the primarily interest was the longer axis (A) of the antenna illumination. Using the parameters for the real system (Chapter 2) the solution for the simple theoretical resolution equation (9) for the horizontal resolution (∆l) or the ability to distinguish between two discreet targets is limited to minimum separation (∆l) of 13.2 cm (5.2 in).

For the survey, two 1.2 m x 0.6 m (4 ft. x 2 ft) UHMW-PE sheets with a thickness of 9.5 mm (3/8 in) cut with a 10˚ triangle removed from each end (Figure A7) were placed between the two large UHMW-PE blocks with the same properties and same field location described in Chapter 2. The top block is 5 cm longer than the bottom block, and flush on the eastern end. Survey lines were taken from North to South, 7.5 cm apart (~ ½ width of the antenna carriage) and perpendicular to the long axis of the air-filled triangle.
$\Delta l = \sqrt{\frac{D\lambda_c}{2}}$ (9)

Figure A5 Horizontal resolution of two discrete targets
Figure A6 Schematic of simple antenna illumination.

\[ A = \frac{\lambda_c}{4} + \frac{D}{\sqrt{\varepsilon_r - 1}} \]

\[ B = \frac{A}{2} \]

Modified from Annan and Cosway 1992
Figure A7 Thin-layer plastic insert for horizontal resolution detection.
The data was exported to EKKO Mapper 3 for processing (default processing included background subtraction, migration, enveloping, and DEWOWing the data) and interpretation. The default (Figure A8) and the non-migrated (Figure A9) processing steps displayed the data well. The dashed line represents where the wedge cut started. The detection of the triangle in the migrated data at ~0.4 m (Figure A8, yellow line) where Δl = 7.6 cm (3 in). The next survey line (~0.45 m) is interpreted as the actual detection of the wedge possibly due to the detection (artifact from software interpolation) in both directions perpendicular to the survey, was measured at ~8.9 cm (3.5 in). The non-migrated processed survey (Figure A9) indicated the same detection limits for the same lines, however at ~0.4 m, the detection appears to be two lines. This was interpreted as the result of edge effects and not the detection of the actual wedge or change in material properties (plastic to air).

When comparing the results from Equation (9) to the actual spatial distance between where the wedge was detected, it is possible detect targets below the A-axis of the ellipse (Figure A6). Additionally, the wedge (0.45 m) is detected 33% below the predicted value, and the edge effects at 42% below the predicted detection limit. Correcting for the source-receiver offset (Figure A5, ray path distance H substituted in for D, or 17.2 cm) and the result is 22.2 cm x 11.1 cm (8.7 in x 4.4 in).

The author suggests this technique may not valid for this model, where the lateral separation is very small (undefined) compared to depth. Equations (11, 12, Annan 2005) are the theoretical complex horizontal resolution equations. Using these equations where Δt = 0.25 ns (suggested the resolution limit between two discreet
targets) $\Delta l = 9.7$ cm (3.8 in). This resulted in a good fit to the confident wedge detection limit, with a difference of 8%. However, the point at which the edge effects are detected is 20% smaller than predicted. Though this complex horizontal resolution equation is a good fit for the actual detection of the material, and further detection is enhanced by the edge effects for the, this may not be applicable for other models or field exercise.

$$\Delta t = \frac{2((D^2 + l^2)^{1/2} - D)}{v}$$  \hfill (11)

$$W = 1 / f_c$$  \hfill (12)
Figure A8 Processed illumination survey. Black lines indicate location of surveys run (from left to right), dashed line indicates origin of wedge, yellow line indicates interpreted location based on processed data. The noise in the lower section of the data is attributed to the overlap of 5 cm of the top block over air.
Figure A9 Non-migrated data.
Appendix 7: Additional Figures
Figure A10 Complete plot of the theoretical reflection amplitude and aperture.
Figure A11 Thin-layer plot of reflection amplitude and aperture.
Figure A12 Reflection Amplitude with salinity. Two distinct trends are observed from 0-20 parts per thousand (ppt) and 20-53 ppt.
Figure A13 Simple schematic of ray antenna ray path.
Figure A14 Simplified reflection and transmission ray paths. Subscript letters indicate relationship between the transmission and reflection amplitude and are not related to the subscripts used anywhere else in this thesis. For a saline water-filled fracture, the reflection from the top of the fracture RA dominate the total signal and TC would travel >2x the fracture thickness increasing attenuation and ultimately contributing little to the reflected signal.
Figure A15 Simplified signal response for a water filled fracture. Distilled water filled fractures components are additive (top) whereas in hypersaline water filled fractures will result in effectively no contribution from the bottom interface.
Vita

Kevin E. Burns was born in West Palm Beach, FL in 1975. After relocating to Texas, he began attending Southwest Texas State University. There he became interested in geology and subsequently transferred to The University of Texas at Austin where he received a BS in Geological Sciences. In 2005, he enrolled at The University of Tennessee and earned a MS in Geology with an emphasis in Geophysics. After a long courtship, Kevin married, Elizabeth M. Jones in 2003. They are happily married with 3 cats, Al Bones, Amy Brownie, and Georgia Kiwi.